**Photon – Electron Interaction**

[working in Natural Gaussian units predominantly] Let’s recall our approximate dipole-dipole interaction insulator model.



where Ωm, formerly known as ωm, tabulates the excitations of the system, and where the polarization vector is:



and for a cubic crystal in the small k limit,



(V being volume, and N the number of lattice sites) which has eigenvectors/values (changing λ → s so can accommodate photon polarization vectors):



And now let’s add an EM field. Like with the metals we’ll start/end up with:



Of course the field is given by (using the CMT phase convention for a, a†):



and here m is the mass of the electron, and Z the number of such mobile electrons on site. The number density operator we’ll take to be a scalar = N/V, which ought to be pretty good for an insulator. The easiest way to get the current density function is from the time-derivative of the polarization function. To get , we must commute with H [apropos **J**p, just the part of H w/o A]. But P commutes with the interaction part, (1/2)KP2, so P will time develop according to just the b†b term. We know how it develops w/r to the free H0­. So we’ll just have:



So differentiating, setting t back to zero, and being more careful about the *density* part, we have:



We can write **J**p(R) in terms of wavevectors,



Then filling this in to just the A part,



It might be best at this point to put J in terms of the K eigenbasis, as we did with P. Then



and,



For our model **K**, we can as well take the ε’s and ξ’s to be parallel to each other, except for the longitudinal part, which the photon doesn’t have. So then we’d have:



where λ runs over just the transverse d.o.f. of **A** and **J**. Doing the same for P in the other part of H, like we had done before in the dipole-dipole insulator file, we come to, all together:



**Feynman Diagram Rules**

So let’s consider the current and photon GF’s.



and of course these are given by:



where,



and,



Now let’s get the unperturbed electron GF. This is:



Note this GF is determined with respect to H0 which includes the P2 term in it. This makes it less trivial to get than usual. Best thing to do is to go back to the insulating folder dipole-dipole interaction files and get it in terms of the polarization-polarization GF. So recall,



So,



So apparently,



which would be, filling in what we found in that file,



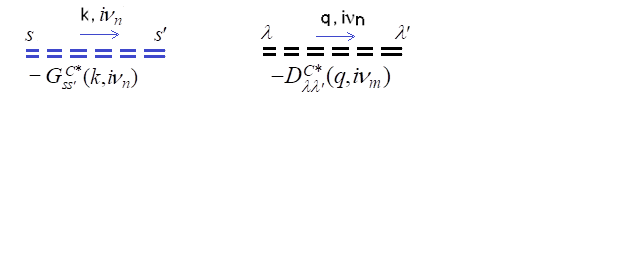
And then the photon GF:



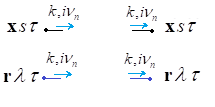
The Feynman rules for single particle GF’s are, basically without any justification…

**Fourier Space Rules**

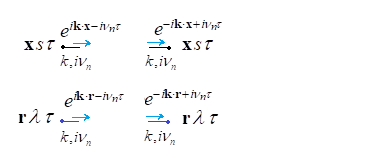
So we have our single GF’s:



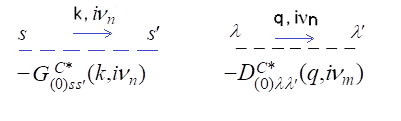
and the external points:



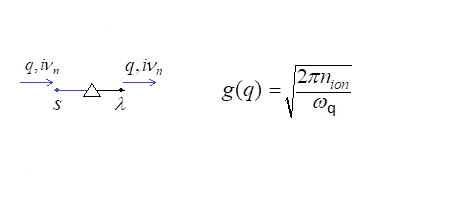
These carry the Fourier transform variable. And if we’re ultimately interested in the real space, time GF, then we must add the following factors to each:



and we connect these with the bare GF’s



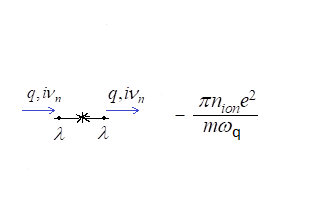
And we associate the Ve-pht with:



(well, we only associate g(q) with it, but I was illustrating the energy conservation that takes place at the vertex) I think that g(q) should be associated with the vertex, and not -ig(q) because extracting a GF from the vertex requires changing J → J† and this comes at a (-) sgn cost, ‘cause:



Moreover, this would involve at least two such interaction terms which carry with them a -i. So we have overall (-)(-i)2 = 1. So that’s why I don’t include the -i. The Vpht-pht interaction would be:



The frequency line is due to fact that Fourier transform of product introduces an extra frequency line – see that example in the Fourier transform file. Also, it might seem that momentum shouldn’t be conserved at the vertex, given the A(-q)A(q) form of the interaction term, but recall that the GF is of form AqAq† [and Aq† = A-q], and so to construct a GF, the A(-q) will have to convert to Aq†. And so then basically, the momentum flowing in will match the momentum flowing out. Same argument applies if we attach to the Aq first. Also, I’ve multiplied the pre-factor by two, because a stray photon line can connect to either end of the vertex, and so there are two possibilities associated with each vertex.

**Topology**

Connect all topologically distinct, fully connected (meaning no vacuum bubbles), diagrams together, associating with each element in the diagram the indicated term. Remember that all energy-momentum labels **k**, ω must be going in the same way w/r to the GF arrow.

**Equal time issues**

None per se´

**Signs/Numerical Factors**

I think every diagram gets a factor of one.

**Sum**

Then sum/integrate over all independent momenta - wavenumbers/energies-frequencies/indices (and polarizations, spins).



Not sure about the (1/V)·Σk deal. Since the unperturbed GF’s are presumably diagonal in ss´, and λλ´, the internal sum over polarizations will just cancel δ functions, with the net effect that whatever polarization we put it in will come out.